Dispersion of active particles in oscillatory Poiseuille flow

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Active particles exhibit complex transport dynamics in flows through confined geometries such as channels or pores. In this work, we employ a generalized Taylor dispersion (GTD) theory to study the long-time dispersion behavior of active Brownian particles (ABPs) in an oscillatory Poiseuille flow within a planar channel. We quantify the time-averaged longitudinal dispersion coefficient as a function of the flow speed, flow oscillation frequency, and particle activity. In the weak-activity limit, asymptotic analysis shows that activity can either enhance or hinder the dispersion compared to the passive case. For arbitrary activity levels, we numerically solve the GTD equations and validate the results with Brownian dynamics simulations. We show that the dispersion coefficient could vary non-monotonically with both the flow speed and particle activity. Furthermore, the dispersion coefficient shows an oscillatory behavior as a function of the flow oscillation frequency, exhibiting distinct minima and maxima at different frequencies. The observed oscillatory dispersion results from the interplay between self-propulsion and oscillatory flow advection—a coupling absent in passive or steady systems. Our results show that time-dependent flows can be used to tune the dispersion of active particles in confinement.

Key words: active matter, colloids, dispersion

1. Introduction

For micron-sized particles, the presence of fluid flow can enhance mass transport due to the interplay between advection and diffusion. A classical example of this coupling effect is Taylor dispersion, where Brownian solutes in pressure-driven flows exhibit enhanced longitudinal dispersion compared to the molecular diffusivity (Taylor 1953, 1954*a*,*b*; Aris 1956). Since the work of Taylor (1953), a generalized Taylor dispersion (GTD) framework has been developed to study a variety of transport phenomena. These include complex geometries, spatial and temporal periodicity, and active (i.e., self-propelled) particle dynamics (Brenner 1980; Shapiro & Brenner 1990; Hill & Bees 2002; Zia & Brady 2010; Alonso-Matilla *et al.* 2019; Peng & Brady 2020; Peng 2024).

Active particles differ from passive solutes in that each unit is capable of self-propulsion (Schweitzer *et al.* 1998; Romanczuk *et al.* 2012). The interplay between self-propulsion

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and fluid flow gives rise to rich and often non-intuitive dynamics that are absent in passive Brownian systems (Romanczuk *et al.* 2012; Bechinger *et al.* 2016; Gomez-Solano *et al.* 2016; Plan *et al.* 2020; Jing *et al.* 2020; Chandragiri *et al.* 2020; Chakraborty *et al.* 2022; Choudhary *et al.* 2022). One example where these dynamics play a crucial role is the transport behavior of microswimmers, which is important for understanding both natural and engineered systems, such as infection by motile bacteria (Siitonen & Nurminen 1992; Lane *et al.* 2005), formation of biofilms (Kim *et al.* 2014; Rusconi *et al.* 2010), drug delivery (Park *et al.* 2017; Lin *et al.* 2021; Díez *et al.* 2021; Sridhar *et al.* 2022), therapeutic treatments (Ghosh *et al.* 2020) and environmental remediation (Soler *et al.* 2013; Urso *et al.* 2023).

Transport of active particles often occur in confined geometries, where Poiseuille flow is a common flow profile, and considerable work has focused on how active matter behaves in such environments (Zöttl & Stark 2012, 2013; Apaza & Sandoval 2016; Junot *et al.* 2019; Mathijssen *et al.* 2019; Chuphal *et al.* 2021; Anand & Singh 2021; Khatri & Burada 2022; Choudhary *et al.* 2022; Walker *et al.* 2022; Ganesh *et al.* 2023; Valani *et al.* 2024). For instance, in channels, active particles exhibit upstream swimming in Poiseuille flow (Kaya & Koser 2012; Kantsler *et al.* 2014; Ezhilan & Saintillan 2015; Omori & Ishikawa 2016). Owing to their upstream motility, *E. coli* introduced downstream causes upstream contamination in initially clean microfluidic channels (Figueroa-Morales *et al.* 2020). Further investigations by Mathijssen *et al.* (2019) on bacterial motion near channel surfaces revealed that *E. coli* engages in distinct rheotaxis regimes depending on the shear rate. With increasing shear, the bacteria transition from upstream swimming to oscillatory rheotaxis, and ultimately to a coexistence of rheotaxis aligned with both positive and negative vorticity directions.

While these studies were primarily focused on steady flows, biologically relevant systems are often governed by time-dependent flow conditions. McDonald (1955) experimentally studied the relationship between pulsatile pressure and blood flow in arteries, analyzing the phasic variations in arterial flow during each cardiac cycle. Inspired by the study of McDonald (1955), Womersley (1955) investigated the velocity, rate of flow, and viscous drag in arteries by considering a time-periodic pressure gradient. The primary factors governing such flows include the pulsatile pressure generated by the heart, the structural and mechanical properties of the vascular walls, and the flow behavior of blood (Secomb 2016).

Early studies on longitudinal dispersion of passive contaminants in oscillatory pressure-driven flows were carried out by Chatwin (1975, 1977). Later, Watson (1983) derived analytical solutions for the long-time effective dispersivity in oscillatory flows within both pipes and rectangular channels. His results showed that the effective dispersivity decreases monotonically with increasing flow frequency. Subsequently, Mazumder & Das (1992) investigated how boundary absorption and heterogeneous reactions influence contaminant dispersion in both steady and oscillatory flows. The significance of such boundary interactions lies in their relevance to processes such as deposition and transport across semi-permeable membranes. More recently, Chu *et al.* (2019) developed a macro-transport theory for two-dimensional flows in a parallel plate channel with alternating shear-free and no-slip regions. They considered both steady and oscillatory flow components to study the transport coefficients of passive particles. Later, they extended their analysis to eccentric annuli (Chu *et al.* 2020) where they showed that the maximum dispersion observed in a time-oscillatory flow can be achieved by applying a slowly oscillating flow in an annulus with large eccentricity. Hettiarachchi *et al.* (2011) used experiments and simulations to show that pulsatile cerebrospinal fluid significantly enhances drug dispersion in the spinal cord relative to no flow.

Although the dispersion of passive particles in oscillatory flows has been widely studied, much less is known about the transport of microswimmers in oscillatory flows. Recently, using experiments and simulations, Caldag & Bees (2025) showed that oscillatory flow can lead to nontrivial dispersion dynamics in gyrotactic swimmers. In this paper, we consider the dispersion of active Brownian particles (ABPs) in time-periodic pressure-driven Poiseuille flow through planar channels. We apply the GTD theory of Peng & Brady (2020), originally developed for ABPs in steady flow, to characterize the long-time longitudinal dispersion of ABPs in oscillatory flow. Due to the time-periodic nature of the flow, an additional time average over one oscillation period is performed to define the time-averaged dispersion coefficient (Chatwin 1975, 1977; Watson 1983). In the weak-swimming limit, characterized by a small swim Péclet number ($Pe_s \ll 1$), we show that the first effect of swimming on longitudinal dispersion appears at $O(Pe_s^2)$. Depending on the flow Péclet number (Pe) and oscillation frequency, the $O(Pe_s^2)$ contribution can be either positive or negative. As such, activity can either enhance or hinder longitudinal dispersion in oscillatory Poiseuille flow compared to passive Brownian particles. For arbitrary swim speeds, numerical solutions of the governing equations are used to characterize the dispersion as a function of the flow speed, swim speed, and oscillation frequency. Numerical results are validated against Brownian dynamics (BD) simulations.

2. Problem formulation

2.1. *The Smoluchowski equation*

We consider the long-time transport behavior of ABPs dispersed in a viscous Newtonian solvent confined between two parallel plates with a separation distance of $2H$. In the dilute limit, we only consider the dynamics of a single ABP. The ABP is assumed to be spherical, and its radius is much smaller than the width of the channel. This allows us to treat the ABP as a 'point' particle. An ABP self-propels with a constant swim speed U_s in a body-fixed swimming direction $\boldsymbol{q}$ ($\boldsymbol{q} \cdot \boldsymbol{q} = 1$). Due to rotational Brownian motion, the orientation vector $\boldsymbol{q}$ undergoes stochastic reorientation. The configuration of an ABP at time t is described by its position vector $\boldsymbol{x}$ and by the orientation vector $\boldsymbol{q}$. We define $P(\boldsymbol{x}, \boldsymbol{q}, t)$ as the probability density function of finding the ABP at position $\boldsymbol{x}$ with orientation $\boldsymbol{q}$ at time t. It satisfies the Smoluchowski equation,

$$\frac{\partial P}{\partial t} + \nabla \cdot \boldsymbol{j}_T + \nabla_R \cdot \boldsymbol{j}_R = 0, \tag{2.1}$$

where $\nabla = \partial/\partial \boldsymbol{x}$ and $\nabla_R = \boldsymbol{q} \times \partial/\partial \boldsymbol{q}$ are the spatial and rotational gradient operators, respectively. In equation (2.1),

$$\boldsymbol{j}_T = U_s \boldsymbol{q} P + \boldsymbol{u}_f P - D_T \nabla P, \tag{2.2}$$

$$\boldsymbol{j}_R = \boldsymbol{\Omega}_f P - D_R \nabla_R P, \tag{2.3}$$

where $\boldsymbol{u}_f$ is the background fluid velocity field, D_T is the translational diffusivity of the ABP, and $\boldsymbol{\Omega}_f = \frac{1}{2} \nabla \times \boldsymbol{u}_f$ is the flow-induced angular velocity. At the channel walls, the no-flux boundary condition is satisfied (Ezhilan & Saintillan 2015; Peng & Brady 2020):

$$\boldsymbol{e}_y \cdot \boldsymbol{j}_T = 0, \quad y = \pm H, \tag{2.4}$$

where $\boldsymbol{e}_y$ is the unit normal to the channel walls. The longitudinal Cartesian coordinate is x and y is the transverse coordinate.

2.2. *Oscillatory Poiseuille flow*

For ease of reference, we provide a brief outline of the flow field derivation. We consider a one-dimensional flow, $\boldsymbol{u}_f = u(y,t)\boldsymbol{e}_x$, driven by a prescribed oscillatory pressure gradient along the channel (Womersley 1955). Here $\boldsymbol{e}_x$ is the unit basis vector in the longitudinal direction. The Navier-Stokes equations reduce to

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}, \tag{2.5}$$

where ρ is the density of the fluid, μ is the dynamic viscosity of the fluid, and the prescribed pressure gradient is given by

$$-\frac{\partial p}{\partial x} = \frac{P_0}{H} \cos(\omega t). \tag{2.6}$$

In equation (2.6), P_0 is a reference pressure and ω is the angular frequency of the actuation. One can show that the solution of equation (2.5) may be written as $u(y,t) = \mathrm{Re}\left[u'(y)e^{i\omega t}\right]$, where

$$u'(y) = \frac{iP_0}{\rho H \omega}\left[-1 + \cosh\left((1+i)\lambda y\right) \operatorname{sech}\left((1+i)\lambda H\right)\right]. \tag{2.7}$$

In equation (2.7), $i = \sqrt{-1}$ is the imaginary unit, $\nu = \mu/\rho$ is the kinematic viscosity of the fluid, and $\lambda = \sqrt{\omega/(2\nu)}$. The viscous length, $1/\lambda = \sqrt{2\nu/\omega}$, sets the scale over which the fluid momentum diffuses during one oscillation cycle of the applied pressure. The operator Re extracts the real part of a complex quantity.

In the zero-frequency limit, $\omega \to 0$, we recover the steady Poiseuille flow as

$$u(y,t) \to \frac{P_0 H}{2\mu}\left(1 - \frac{y^2}{H^2}\right). \tag{2.8}$$

For convenience, we define the characteristic flow speed $U_f = P_0 H/(2\mu)$. Using this, we rewrite equation (2.7) as

$$u'(y) = \frac{iU_f}{(\lambda H)^2}\left[-1 + \cosh\left((1+i)\lambda y\right) \operatorname{sech}\left((1+i)\lambda H\right)\right], \tag{2.9}$$

The angular velocity $\Omega_f(y,t) = \mathrm{Re}\left[\Omega' e^{i\omega t}\right]$, where

$$\Omega' = -\frac{1}{2}\frac{\partial u'}{\partial y} = \frac{(1-i)U_f}{2\lambda H^2} \sinh\left((1+i)\lambda y\right) \operatorname{sech}\left((1+i)\lambda H\right). \tag{2.10}$$

2.3. *Generalized Taylor dispersion theory*

Taking the zeroth orientational moment of equation (2.1) gives the governing equation for the number density,

$$\frac{\partial n}{\partial t} + \nabla \cdot \left(\boldsymbol{u}_f n + U_s \boldsymbol{m} - D_T \nabla n\right) = 0, \tag{2.11}$$

where $n = \int_{\mathbb{S}} P \mathrm{d}\boldsymbol{q}$ is the number density, and $\boldsymbol{m} = \int_{\mathbb{S}} \boldsymbol{q} P \mathrm{d}\boldsymbol{q}$ is the first moment, or polar order. Here $\mathbb{S} = \{\boldsymbol{q} \,|\, \boldsymbol{q}\cdot\boldsymbol{q} = 1\}$ denotes the unit sphere of orientations. Since the channel is unbounded in the x direction, it is convenient to work in Fourier space. To derive a long-time effective transport equation, we first define the Fourier transform of a function $f(x)$ as $\hat{f}(k) = \int e^{-ikx} f(x) \mathrm{d}x$, where k is the wavenumber. Following Peng & Brady (2020), one can show that

$$\frac{\partial \overline{n}}{\partial t} + k^2 D_T \overline{n} + ik\left(\overline{u(y,t)\hat{n}} + U_s \overline{\hat{m}_x}\right) = 0, \tag{2.12}$$

where we have made use of the no-flux condition, and an overhead bar denotes the cross-sectional average,

$$\overline{n}(k,t) = \frac{1}{2H}\int_{-H}^{H} \hat{n}(k,y,t)\mathrm{d}y. \tag{2.13}$$

In equation (2.12), $\hat{m}_x = \boldsymbol{e}_x \cdot \hat{\boldsymbol{m}}$ is the polar order in the x direction in Fourier space.

Introducing the non-dimensional density or structure function $\hat{G}$ such that $\hat{P}(k,y,\boldsymbol{q},t) = \overline{n}(k,t)\hat{G}(k,y,\boldsymbol{q},t)$ and the small wavenumber expansion $\hat{G} = g(y,\boldsymbol{q},t) + ik\,b(y,\boldsymbol{q},t) + O(k^2)$, we obtain

$$\frac{\partial \overline{n}}{\partial t} + ikU^{\mathrm{eff}}\overline{n} + k^2 D^{\mathrm{eff}}\overline{n} + O(k^3) = 0, \tag{2.14}$$

where the effective drift and the effective longitudinal dispersivity are given by, respectively,

$$U^{\mathrm{eff}} = U^{\mathrm{eff}}(t) = U_s\overline{m_x^0} + \overline{un^0}, \tag{2.15}$$

$$D^{\mathrm{eff}} = D^{\mathrm{eff}}(t) = D_T - U_s\overline{\tilde{m}_x} - \overline{u\tilde{n}}. \tag{2.16}$$

In the small-wavenumber expansion, g is the average field and b is the displacement (or fluctuating) field. Note that b has units of length, e.g., displacement. We emphasize that terms of order k^3 and higher do not contribute to either the drift or the dispersion coefficient. The orientational moments in (2.15) are given by

$$n^0 = \int_{\mathbb{S}} g\mathrm{d}\boldsymbol{q}, \quad \text{and} \quad \boldsymbol{m}^0 = \int_{\mathbb{S}} \boldsymbol{q}g\mathrm{d}\boldsymbol{q}. \tag{2.17}$$

Similarly, in (2.16), we have

$$\tilde{n} = \int_{\mathbb{S}} b\mathrm{d}\boldsymbol{q}, \quad \text{and} \quad \tilde{\boldsymbol{m}} = \int_{\mathbb{S}} \boldsymbol{q}b\mathrm{d}\boldsymbol{q}. \tag{2.18}$$

Different from the constant transport coefficients in Peng & Brady (2020), the long-time transport coefficients in (2.15) and (2.16) are time-dependent due to the oscillatory flow.

The governing equations and boundary conditions for g and b are derived in Peng & Brady (2020). For the average field, we have

$$\frac{\partial g}{\partial t} + \frac{\partial}{\partial y}\left(U_s q_y g - D_T\frac{\partial g}{\partial y}\right) + \nabla_R\cdot\left(\boldsymbol{\Omega}_f g - D_R\nabla_R g\right) = 0, \tag{2.19}$$

and

$$U_s q_y g - D_T\frac{\partial g}{\partial y} = 0, \quad y = \pm H. \tag{2.20}$$

The displacement field is governed by

$$\frac{\partial b}{\partial t} + \frac{\partial}{\partial y}\left(U_s q_y b - D_T\frac{\partial b}{\partial y}\right) + \nabla_R\cdot\left(\boldsymbol{\Omega}_f b - D_R\nabla_R b\right) = \left(U^{\mathrm{eff}} - u - U_s q_x\right) g, \tag{2.21}$$

$$U_s q_y b - D_T\frac{\partial b}{\partial y} = 0, \quad y = \pm H. \tag{2.22}$$

Noting that

$$\frac{1}{2H}\int_{-H}^{H}\mathrm{d}y\int_{\mathbb{S}}\hat{G}\mathrm{d}\boldsymbol{q} = 1, \tag{2.23}$$

we have

$$\frac{1}{2H}\int_{-H}^{H}\mathrm{d}y\int_{\mathbb{S}} g\mathrm{d}\boldsymbol{q} = 1, \quad \text{and} \quad \frac{1}{2H}\int_{-H}^{H}\mathrm{d}y\int_{\mathbb{S}} b\mathrm{d}\boldsymbol{q} = 0. \tag{2.24}$$

2.4. *Non-dimensionalization*

We scale lengths with the channel half-width H and the time with the reorientation time τ_R. The system is governed by five non-dimensional parameters:

$$Pe = \frac{U_f \tau_R}{H}, \quad Pe_s = \frac{U_s \tau_R}{H} = \frac{\ell}{H}, \quad \gamma = \frac{\sqrt{D_T \tau_R}}{H} = \frac{\delta}{H}, \tag{2.25a}$$

$$\chi = \omega\, \tau_R, \quad \kappa = \lambda H = \sqrt{\omega/(2\nu)} H. \tag{2.25b}$$

where Pe is the flow Péclet number that compares the reorientation time τ_R with the flow timescale H/U_f, Pe_s is the swim Péclet number that compares the reorientation time with the swim timescale H/U_s, γ is a non-dimensional measure of the microscopic length $\delta = \sqrt{D_T \tau_R}$, χ is the non-dimensional flow frequency, and κ compares the length scale $1/\lambda$ with the channel half-width H. The microscopic length δ characterizes the distance a particle travels by translational diffusion over the timescale defined by τ_R. The swim Péclet number can be viewed as a comparison between the persistence length, $\ell = U_s \tau_R$, and the channel half-width. Since both χ and κ contains ω, it is useful to introduce the non-dimensional parameter

$$\alpha = \frac{\chi}{\kappa^2} = \frac{2\nu\tau_R}{H}, \tag{2.26}$$

when analyzing the effect of flow frequency ω on dispersion behavior. With this, varying the dimensional frequency ω corresponds to changing χ while keeping α constant.

The non-dimensional form of equation (2.19) is

$$\frac{\partial g}{\partial t^*} + \frac{\partial}{\partial y^*}\left(Pe_s q_y g - \gamma^2 \frac{\partial g}{\partial y^*}\right) + \frac{\partial}{\partial \phi}\left(\Omega_f^* g - \frac{\partial g}{\partial \phi}\right) = 0, \tag{2.27}$$

where we have used the parametrization $\boldsymbol{q} = \cos\phi\, \boldsymbol{e}_x + \sin\phi\, \boldsymbol{e}_y$ with $\phi \in [0, 2\pi)$ being the orientation angle, $y^* \in [-1, 1]$, and we have used the superscript '$*$' to denote dimensionless quantities. That is, $t^* = t/\tau_R$, $y^* = y/H$, and $\Omega_f^* = \Omega_f \tau_R = \mathrm{Re}\left[\Omega'^* e^{i\chi t^*}\right]$, where

$$\Omega'^* = \Omega' \tau_R = \frac{(1-i)Pe}{2\kappa} \sinh\left((1+i)\kappa y^*\right) \operatorname{sech}\left((1+i)\kappa\right). \tag{2.28}$$

The superscript on g is suppressed since g is non-dimensional. With the solution of g, we can obtain the non-dimensional drift via

$$U^{\mathrm{eff}*}(t^*) = U^{\mathrm{eff}} \tau_R / H = Pe_s \overline{m_x^0} + \overline{u^* n^0}. \tag{2.29}$$

Similarly, we may write the non-dimensional form of equation (2.21) as

$$\frac{\partial b^*}{\partial t^*} + \frac{\partial}{\partial y^*}\left(Pe_s q_y b^* - \gamma^2 \frac{\partial b^*}{\partial y^*}\right) + \frac{\partial}{\partial \phi}\left(\Omega_f^* b^* - \frac{\partial b^*}{\partial \phi}\right) = \left(U^{\mathrm{eff}*} - u^* - Pe_s q_x\right) g, \tag{2.30}$$

where $b^* = b/H$, and $u^* = u\tau_R/H = \mathrm{Re}[u'^* e^{i\chi t^*}]$. The complex flow amplitude is given by

$$u'^* = \frac{iPe}{\kappa^2}\left[-1 + \cosh\left((1+i)\kappa y^*\right) \operatorname{sech}\left((1+i)\kappa\right)\right]. \tag{2.31}$$

To characterize the dispersion of active particles in an oscillatory Poiseuille flow, we compare the effective dispersion coefficient with the translational diffusivity. Using equation (2.16), we have

$$D^{\mathrm{eff}*} = \frac{D^{\mathrm{eff}}}{D_T} = 1 - \frac{Pe_s}{\gamma^2}\overline{\tilde{m}_x^*} - \frac{1}{\gamma^2}\overline{u^* \tilde{n}^*}. \tag{2.32}$$

If $Pe = 0$, or $U_f = 0$, the problem reduces to that of diffusion of ABPs in a flat channel

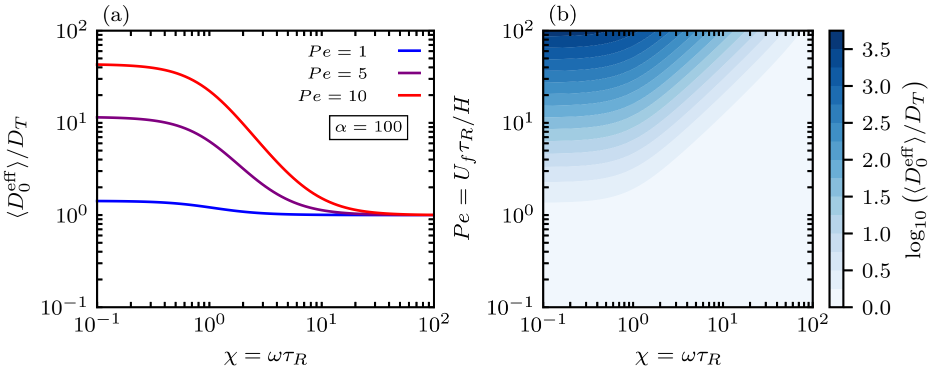

FIGURE 1. (a) Plots of the non-dimensional time-averaged effective dispersivity ($\langle D_0^{\text{eff}} \rangle / D_T$) as a function of χ. (b) Contour plot of the logarithm of $\langle D_0^{\text{eff}} \rangle / D_T$ as a function of Pe and χ. For all results shown, $\alpha = 100$, and $\gamma^2 = 0.1$.

without flow. In this case, we have $D^{\text{eff}} = D_{\text{nf}}^{\text{eff}} = D_T + D^{\text{swim}}$, where $D^{\text{swim}} = U_s^2 \tau_R / 2$ in 2D (Berg 1993), and $D_{\text{nf}}^{\text{eff}}$ is the effective dispersivity without flow. In non-dimensional form, we have

$$\frac{D_{\text{nf}}^{\text{eff}}}{D_T} = 1 + \frac{Pe_s^2}{2\gamma^2}. \tag{2.33}$$

For an oscillatory Poiseuille flow, D^{eff} after the initial transients becomes a periodic function of time. At long times, we define the time-averaged effective dispersion coefficient as

$$\langle D^{\text{eff}*} \rangle = \lim_{t' \to \infty} \frac{1}{T} \int_{t'}^{t'+T} D^{\text{eff}*}(t^*) \mathrm{d}t^*, \tag{2.34}$$

where $T = 2\pi/\chi$ is the period of the flow oscillation. Similarly, one can define the time-averaged effective drift as $\langle U^{\text{eff}*} \rangle$.

3. Weak-swimming asymptotic analysis

In the weak-swimming limit, characterized by $Pe_s \ll 1$, we pose regular expansions for the fields and transport coefficients:

$$g = g_0 + Pe_s\, g_1 + Pe_s^2\, g_2 + \cdots, \tag{3.1}$$

$$b^* = b_0^* + Pe_s\, b_1^* + Pe_s^2\, b_2^* + \cdots, \tag{3.2}$$

$$U^{\text{eff}*} = U_0^{\text{eff}*} + Pe_s\, U_1^{\text{eff}*} + Pe_s^2\, U_2^{\text{eff}*} + \cdots, \tag{3.3}$$

$$D^{\text{eff}*} = D_0^{\text{eff}*} + Pe_s\, D_1^{\text{eff}*} + Pe_s^2\, D_2^{\text{eff}*} + \cdots. \tag{3.4}$$

3.1. *Passive Brownian particles*

At $O(1)$, the particle is passive and the average field is given by $g_0 \equiv 1/(2\pi)$. This means that the number density across the channel is uniform. As a result, the effective drift at $O(1)$ is given by $U_0^{\text{eff}*} = \overline{u^*}$, which vanishes upon time-averaging.

The displacement field at $O(1)$ admits a solution of the form $b_0^* = \mathrm{Re}[A_0'(y^*) e^{i\chi t^*}/(2\pi)]$, where the solution to A_0' is provided in appendix A. The instantaneous effective dispersion coefficient at $O(1)$ after initial transients is given by

$$D_0^{\text{eff}*}(t^*) = 1 - \frac{1}{2\gamma^2} \int_{-1}^{1} u^* \,\mathrm{Re}\left[A_0' e^{i\chi t^*}\right] \mathrm{d}y^*. \tag{3.5}$$

An analytical expression for the effective dispersion coefficient was derived by Watson (1983), given by

$$\langle D_0^{\mathrm{eff}*}\rangle = 1 + \frac{Pe^2}{\kappa^2}\frac{\cosh(2\kappa) - \cos(2\kappa)}{\cosh(2\kappa) + \cos(2\kappa)}\frac{\iota(2\kappa) - \iota(\sqrt{2\chi}/\gamma)}{\chi^2 - 4\gamma^4\kappa^4}, \tag{3.6}$$

where

$$\iota(a) = \frac{\sinh(a) - \sin(a)}{a(\cosh(a) - \cos(a))}. \tag{3.7}$$

In figure 1, we plot the passive dispersivity ($\langle D_0^{\mathrm{eff}}\rangle / D_T$), given in (3.6), as a function of χ and Pe. Since α is held fixed, increasing χ corresponds to increasing the dimensional frequency. In the low frequency limit, we have $\langle D_0^{\mathrm{eff}}\rangle / D_T \to 1 + 4Pe^2/(945\gamma^4)$ as $\chi \to 0$. For a steady Poiseuille flow of the same amplitude, the long-time dispersion coefficient $D_0^{\mathrm{eff}}/D_T = 1 + 8Pe^2/(945\gamma^4)$. As is well known, in oscillatory flow, $(\langle D_0^{\mathrm{eff}}\rangle - D_T)/D_T$ approaches half of its steady value as $\chi \to 0$ (Aris 1960; Bowden 1965; Van den Broeck 1982; Watson 1983; Ng 2006; Chu *et al.* 2019, 2020). On the other hand, as $\chi \to \infty$, $\langle D_0^{\mathrm{eff}}\rangle / D_T \to 1$ regardless of Pe [see figure 1(a)]. In this high-frequency limit, shear-induced dispersion vanishes due to the rapid flow oscillations. For low and intermediate frequencies, $\langle D_0^{\mathrm{eff}}\rangle$ increases with Pe, as is consistent with Taylor dispersion. Overall, $\langle D_0^{\mathrm{eff}}\rangle$ decreases monotonically with increasing frequency until it reaches the high-frequency limit. In figure 1(b), we plot the same analytical expression given in equation (3.6) in a contour plot as a function of both χ and Pe.

3.2. *First order*

At $O(Pe_s)$, the average field is governed by

$$\frac{\partial g_1}{\partial t^*} + \frac{\partial}{\partial y^*}\left(-\gamma^2\frac{\partial g_1}{\partial y^*}\right) + \frac{\partial}{\partial \phi}\left(\Omega_f^* g_1 - \frac{\partial g_1}{\partial \phi}\right) = -q_y\frac{\partial g_0}{\partial y^*}, \tag{3.8a}$$

$$\gamma^2\frac{\partial g_1}{\partial y^*} = q_y g_0, \quad \text{at} \quad y^* = \pm 1, \tag{3.8b}$$

$$\int_{-1}^{1} \mathrm{d}y^* \int_{\mathbb{S}} g_1 \mathrm{d}\boldsymbol{q} = 0. \tag{3.8c}$$

Assuming a solution of the form $g_1 = A_1(y^*, t^*)\cos\phi + B_1(y^*, t^*)\sin\phi$, we obtain

$$\frac{\partial A_1}{\partial t^*} - \gamma^2\frac{\partial^2 A_1}{\partial y^{*2}} + \Omega_f^* B_1 + A_1 = 0, \tag{3.9a}$$

$$\frac{\partial B_1}{\partial t^*} - \gamma^2\frac{\partial^2 B_1}{\partial y^{*2}} - \Omega_f^* A_1 + B_1 = 0, \tag{3.9b}$$

$$\frac{\partial A_1}{\partial y^*} = 0, \quad \text{and} \quad \frac{\partial B_1}{\partial y^*} = \frac{1}{2\pi\gamma^2}, \quad \text{at} \quad y^* = \pm 1. \tag{3.9c}$$

The instantaneous effective drift at this order $U_1^{\mathrm{eff}*}$ vanishes.

The displacement filed at $O(Pe_s)$ is governed by

$$\frac{\partial b_1^*}{\partial t^*} + \frac{\partial}{\partial y^*}\left(-\gamma^2\frac{\partial b_1^*}{\partial y^*}\right) + \frac{\partial}{\partial \phi}\left(\Omega_f^* b_1^* - \frac{\partial b_1^*}{\partial \phi}\right) = -q_y\frac{\partial b_0^*}{\partial y^*} + \left(U_0^{\mathrm{eff}*} - u^*\right) g_1 + \left(U_1^{\mathrm{eff}*} - q_x\right) g_0, \tag{3.10a}$$

$$\gamma^2\frac{\partial b_1^*}{\partial y^*} = q_y b_0^*, \quad \text{at} \quad y^* = \pm 1, \tag{3.10b}$$

$$\int_{-1}^{1} \mathrm{d}y^* \int_{\mathbb{S}} b_1^* \mathrm{d}\boldsymbol{q} = 0, \tag{3.10c}$$

which admits a solution of the form $b_1^* = A_2(y^*, t^*)\cos\phi + B_2(y^*, t^*)\sin\phi$. Inserting this form into equation (3.10), we obtain

$$\frac{\partial A_2}{\partial t^*} - \gamma^2 \frac{\partial^2 A_2}{\partial y^{*2}} + \mathit{\Omega}_f^* B_2 + A_2 = \left(U_0^{\mathrm{eff}*} - u^*\right) A_1 - g_0, \tag{3.11a}$$

$$\frac{\partial B_2}{\partial t^*} - \gamma^2 \frac{\partial^2 B_2}{\partial y^{*2}} - \mathit{\Omega}_f^* A_2 + B_2 = -\frac{\partial b_0^*}{\partial y^*} + \left(U_0^{\mathrm{eff}*} - u^*\right) B_1, \tag{3.11b}$$

$$\frac{\partial A_2}{\partial y^*} = 0, \quad \text{and} \quad \frac{\partial B_2}{\partial y^*} = \frac{b_0^*}{\gamma^2}, \quad \text{at} \quad y^* = \pm 1. \tag{3.11c}$$

The effective longitudinal dispersivity at $O(Pe_s)$ vanishes,

$$D_1^{\mathrm{eff}*} = -\frac{1}{2\gamma^2}\int_{-1}^{1} \mathrm{d}y^* \int_{\mathbb{S}} (u^* b_1^* + q_x b_0^*) \mathrm{d}\boldsymbol{q} = 0. \tag{3.12}$$

3.3. *Second order*

At $O(Pe_s^2)$, the average field is governed by

$$\frac{\partial g_2}{\partial t^*} + \frac{\partial}{\partial y^*}\left(q_y g_1 - \gamma^2 \frac{\partial g_2}{\partial y^*}\right) + \frac{\partial}{\partial \phi}\left(\mathit{\Omega}_f^* g_2 - \frac{\partial g_2}{\partial \phi}\right) = 0, \tag{3.13a}$$

$$\gamma^2 \frac{\partial g_2}{\partial y^*} = q_y g_1 \quad \text{at} \quad y^* = \pm 1, \tag{3.13b}$$

$$\int_{-1}^{1} \mathrm{d}y^* \int_{\mathbb{S}} g_2 \mathrm{d}\boldsymbol{q} = 0. \tag{3.13c}$$

We propose a solution of the form,

$$g_2 = K_1(y^*, t^*) + C_1(y^*, t^*)\cos 2\phi + D_1(y^*, t^*)\sin 2\phi. \tag{3.14}$$

The displacement filed at $O(Pe_s^2)$ is governed by

$$\frac{\partial b_2^*}{\partial t^*} + \frac{\partial}{\partial y^*}\left(q_y b_1^* - \gamma^2 \frac{\partial b_2^*}{\partial y^*}\right) + \frac{\partial}{\partial \phi}\left(\mathit{\Omega}_f^* b_2^* - \frac{\partial b_2^*}{\partial \phi}\right)$$
$$= \left(U_0^{\mathrm{eff}*} - u^*\right) g_2 + \left(U_1^{\mathrm{eff}*} - q_x\right) g_1 + U_2^{\mathrm{eff}*} g_0, \tag{3.15a}$$

$$\gamma^2 \frac{\partial b_2^*}{\partial y^*} = q_y b_1^* \quad \text{at} \quad y^* = \pm 1, \tag{3.15b}$$

$$\int_{-1}^{1} \mathrm{d}y^* \int_{\mathbb{S}} b_2^* \mathrm{d}\boldsymbol{q} = 0. \tag{3.15c}$$

We assume a solution for b_2^*,

$$b_2^* = K_2(y^*, t^*) + C_2(y^*, t^*)\cos 2\phi + D_2(y^*, t^*)\sin 2\phi. \tag{3.16}$$

One can show that $\langle U_2^{\mathrm{eff}*}\rangle = 0$, and

$$D_2^{\mathrm{eff}*} = -\frac{1}{2\gamma^2}\int_{-1}^{1} \mathrm{d}y^* \int_{\mathbb{S}} (u^* b_2^* + q_x b_1^*) \mathrm{d}\boldsymbol{q} = -\frac{\pi}{2\gamma^2}\int_{-1}^{1} (2u^* K_2 + A_2)\, \mathrm{d}y^*. \tag{3.17}$$

To obtain $D_2^{\mathrm{eff}*}$, one needs to solve for K_2. The relevant equations are given by

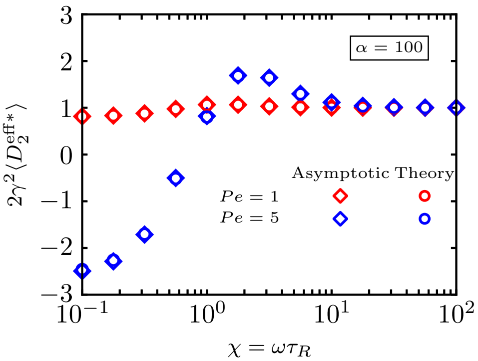

FIGURE 2. The $O(Pe_s^2)$ dispersivity as a function of χ. For all results, $\alpha = 100$, and $\gamma^2 = 0.1$. Circles denote results obtained from the numerical solutions of the full GTD theory for $Pe_s = 0.1$. Diamonds denote results from the asymptotic analysis.

$$\frac{\partial K_1}{\partial t^*} - \gamma^2 \frac{\partial^2 K_1}{\partial y^{*2}} + \frac{1}{2}\frac{\partial B_1}{\partial y^*} = 0, \tag{3.18a}$$

$$\frac{\partial K_2}{\partial t^*} + \left[\frac{1}{2}\frac{\partial B_2}{\partial y^*} - \gamma^2 \frac{\partial^2 K_2}{\partial y^{*2}}\right] = U_2^{\mathrm{eff}*} g_0 - \frac{1}{2}A_1 + \left(U_0^{\mathrm{eff}*} - u^*\right) K_1, \tag{3.18b}$$

$$\frac{\partial K_1}{\partial y^*} = \frac{1}{2\gamma^2}B_1, \quad \text{and} \quad \frac{\partial K_2}{\partial y^*} = \frac{1}{2\gamma^2}B_2 \quad \text{at} \quad y^* = \pm 1. \tag{3.18c}$$

We solve equations (3.9), (3.11) and (3.18) using a Chebyshev collocation method. For time evolution, we use the Crank-Nicolson method. At long times, the time-averaged dispersion coefficient, $\langle D_2^{\mathrm{eff}*}\rangle$, is obtained via numerical integration over one oscillation period. We also solve the full GTD theory by solving equations (2.27) and (2.30) numerically (see appendix D). To extract an approximation of $D_2^{\mathrm{eff}*}$ from the full solution, denoted as $\tilde{D}_2^{\mathrm{eff}*}$, we use the relation $\langle \tilde{D}_2^{\mathrm{eff}*}\rangle = \left(\langle D^{\mathrm{eff}*}\rangle - \langle D_0^{\mathrm{eff}*}\rangle\right)/Pe_s^2$. Here, $\langle D_0^{\mathrm{eff}*}\rangle$ is the analytical solution for passive particles from equation (3.6), and the full simulation is performed with $Pe_s = 0.1$.

In figure 2, we plot $\langle D_2^{\mathrm{eff}*}\rangle$ as a function of χ. The asymptotic results (diamonds) are compared with numerical solutions (circles) of the full GTD theory (see appendix D). As in the passive case (see figure 1), the shear-induced dispersion vanishes in the high-frequency limit. From (2.33), we have $\langle D_2^{\mathrm{eff}*}\rangle \to 1/(2\gamma^2)$ as $\chi \to \infty$. Indeed, figure 2 shows that $2\gamma^2 \langle D_2^{\mathrm{eff}*}\rangle$ approaches unity in the high-frequency limit.

Overall, $\langle D_2^{\mathrm{eff}*}\rangle$ can be either positive or negative depending on Pe and χ. This means that activity can either enhance or hinder the longitudinal dispersion in an oscillatory flow compared to the passive case. In particular, a reduction in the dispersion ($\langle D_2^{\mathrm{eff}*}\rangle < 0$) occurs in the low-frequency regime when Pe is sufficiently large (e.g., $Pe = 5$; blue markers). This reduction can be attributed to shear-reduced swim diffusion (Peng & Brady 2020), which becomes prominent for sufficiently strong shear. For $Pe = 1$, $\langle D_2^{\mathrm{eff}*}\rangle > 0$ for all values of χ. For $Pe = 5$, $\langle D_2^{\mathrm{eff}*}\rangle$ can be either positive or negative depending on χ. There exists an optimal frequency at which the enhancement in dispersion is maximized.

In figure 3, we compare $\langle D^{\mathrm{eff}}\rangle/D_T$ from the two-term asymptotic solution (solid lines), $\langle D_0^{\mathrm{eff}*}\rangle + Pe_s^2\langle D_2^{\mathrm{eff}*}\rangle$, with the numerical solutions (circles) of the full GTD theory. In figure 3(a), for $Pe = 1$, the two-term asymptotic solution (solid line) agrees with the

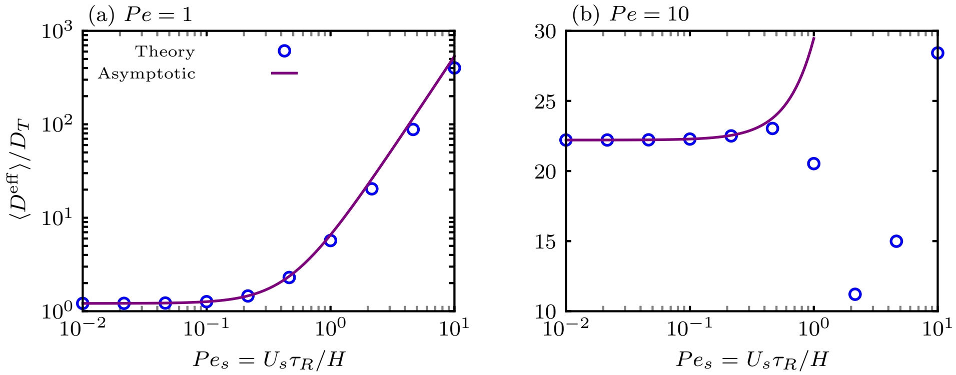

FIGURE 3. Plots of $\langle D^{\text{eff}}\rangle/D_T$ as a function of Pe_s for (a) $Pe = 1$, and (b) $Pe = 10$. The solid lines denote the two-term asymptotic solution, $\langle D_0^{\text{eff}*}\rangle + Pe_s^2\langle D_2^{\text{eff}*}\rangle$. Circles are numerical solutions of the full GTD theory. For all results shown, $\chi = 1$, $\gamma^2 = 0.1$, and $\kappa = 0.1$.

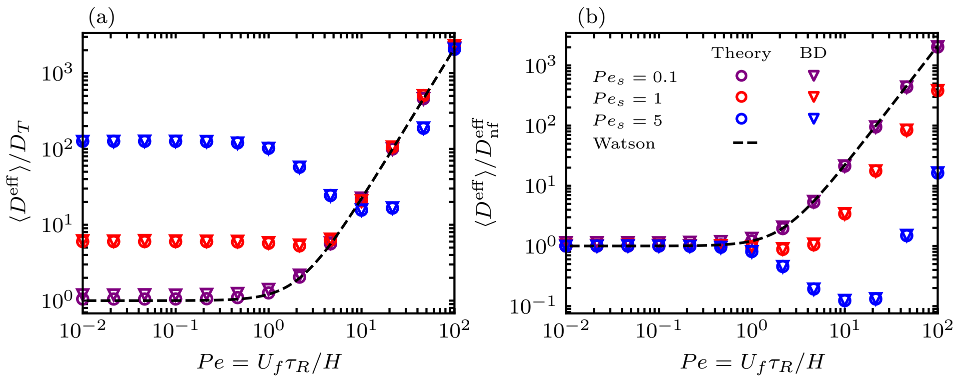

FIGURE 4. (a) Plots of $\langle D^{\text{eff}}\rangle/D_T$ as a function of Pe for several values of Pe_s. (b) Plots of $\langle D^{\text{eff}}\rangle/D_{\text{nf}}^{\text{eff}}$ as a function of Pe for several values of Pe_s. Circles represent solutions of the full GTD theory, and triangles denote results from BD simulations. The dashed line represents the passive ($Pe_s = 0$) results. For all results, $\chi = 1, \gamma^2 = 0.1$, and $\kappa = 0.1$.

full GTD theory (circles) well beyond its formal regime of validity, i.e., $Pe_s \ll 1$. In figure 3(b), for a stronger flow ($Pe = 10$), the full GTD results (circles) show that $\langle D^{\text{eff}}\rangle/D_T$ varies non-monotonically with increasing Pe_s. As Pe_s increases beyond the weak-swimming regime, the effective dispersivity decreases due to shear-reduced swim diffusion. The effective dispersivity increases again when activity (Pe_s) is sufficiently high. This behavior is not captured by the asymptotic solution (solid line), which is valid only in the weak-swimming limit.

4. Dispersivity in the finite activity regime

To characterize the general behavior of the effective dispersion coefficient, we resort to numerical solutions of the full GTD theory (see appendix D). The GTD equations are evolved over time. Numerical solutions of the GTD theory are compared to results obtained from BD simulations (see appendix C).

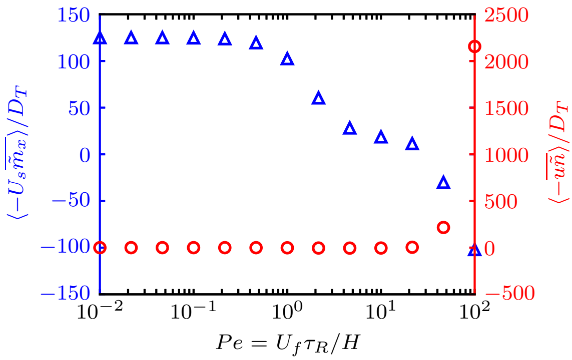

FIGURE 5. Plots of the two contributions to $\langle D^{\text{eff}}\rangle / D_T$, $\langle -U_s\overline{\tilde{m}_x}\rangle / D_T$ and $\langle -\overline{u\tilde{n}}\rangle / D_T$ as a function of Pe for $Pe_s = 5$. Blue triangles represent $\langle -U_s\overline{\tilde{m}_x}\rangle / D_T$, and red circles represent $\langle -\overline{u\tilde{n}}\rangle / D_T$. All results are obtained by solving the full GTD theory with $\chi = 1$, $\gamma^2 = 0.1$, and $\kappa = 0.1$.

4.1. *Competition between flow advection and particle activity*

In this section, we examine the dispersion behavior of ABPs for a given flow oscillation frequency, $\chi = 1$. With this fixed frequency, the dispersion is qualitatively similar to that considered by Peng & Brady (2020) for a steady Poiseuille flow. In figure 4(a), we plot $\langle D^{\text{eff}}\rangle / D_T$ as a function of Pe for different values of Pe_s. As $Pe \to 0$, we recover the dispersion coefficient in the absence of flow, $D^{\text{eff}}_{\text{nf}}$. Since $D^{\text{eff}}_{\text{nf}} / D_T = 1 + Pe_s^2/(2\gamma^2)$, the low-$Pe$ plateau in the dispersion coefficient increases with activity (Pe_s). On the other hand, as $Pe \to \infty$, the flow speed dominates over the swim speed. In this regime, the effective dispersion coefficient converges to the passive result (dashed line), regardless of activity. For higher activity (e.g., $Pe_s = 5$; blue markers), a large flow amplitude (Pe) is required for the dispersion coefficient to approach the passive result. For $Pe_s = 0.1$ (purple markers) and $Pe_s = 1$ (red markers), the swimming effects are largely dominated by the Taylor dispersion component. When activity is sufficiently high (e.g., $Pe_s = 5$; blue markers), the dispersion coefficient varies non-monotonically with increasing flow amplitude. The reduction in $\langle D^{\text{eff}}\rangle$ for intermediate flow amplitudes are due to the shear-reduced swim diffusion (see also § 3.3).

In figure 4(b), we replot the data shown in figure 4(a) using a different scaling—$\langle D^{\text{eff}}\rangle / D^{\text{eff}}_{\text{nf}}$ instead of $\langle D^{\text{eff}}\rangle / D_T$. By scaling the effective dispersion with the no-flow dispersion coefficient, all curves collapse in the low-Pe limit. Conversely, the rescaled dispersion coefficient approaches different values in the large-Pe limit.

To visualize the non-monotonic behavior of $\langle D^{\text{eff}}\rangle / D_T$, we plot its two contributions, $\langle -U_s\overline{\tilde{m}_x}\rangle / D_T$ and $\langle -\overline{u\tilde{n}}\rangle / D_T$ as functions of Pe in figure 5. In the intermediate Pe regime, $\langle -U_s\overline{\tilde{m}_x}\rangle / D_T$ decreases with increasing Pe and even becomes negative at high Pe. In contrast, $\langle -\overline{u\tilde{n}}\rangle / D_T$ increases with Pe. For sufficiently large Pe, the Taylor component dominates over the swim contribution. The competition between these two contributions gives rise to the observed non-monotonicity in the effective dispersivity, as shown in figure 4. We emphasize that these two contributions are not independent; therefore, each individual term should not be interpreted as a dispersion coefficient.

4.2. *Effect of oscillation frequency*

We now consider the effect of flow oscillation frequency on the effective longitudinal dispersion. In figure 6, we plot $\langle D^{\text{eff}}\rangle / D^{\text{eff}}_{\text{nf}}$ as a function of χ for different values of Pe

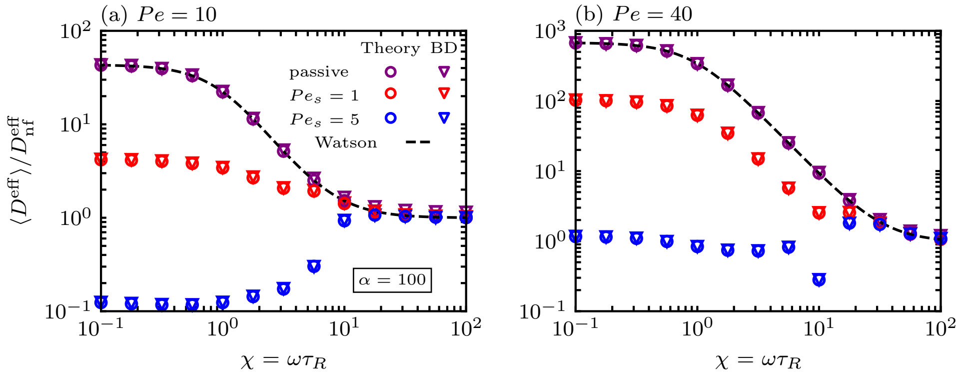

FIGURE 6. Plots of $\langle D^{\text{eff}}\rangle / D^{\text{eff}}_{\text{nf}}$ versus χ for different values of Pe_s, shown for (a) $Pe = 10$, and (b) $Pe = 40$. For all results shown, $\alpha = 100$, and $\gamma^2 = 0.1$. Circles denote results obtained from numerical solutions of the full GTD theory, and triangles represent results from BD simulations. The dashed line represents the passive ($Pe_s = 0$) results.

and Pe_s. Circles denote numerical solutions of the GTD theory, while triangles represent results from BD simulations. The dashed line corresponds to the passive case (i.e., $Pe_s = 0$), as determined by Watson (1983). Figures 6(a) and 6(b) correspond to $Pe = 10$ and $Pe = 40$, respectively. To isolate the effect of the oscillation frequency, we fix α in this section. With α fixed, we note that κ depends explicitly on χ (i.e., $\kappa^2 = \chi/\alpha$).

In the high-frequency limit, one can show that the flow vanishes at leading order (see appendix B for the asymptotic analysis). Effectively, the high-frequency limit is equivalent to the no-flow case. Therefore, $\left\langle D^{\text{eff}}\right\rangle \to D^{\text{eff}}_{\text{nf}}$ as $\chi \to \infty$, regardless of Pe_s or Pe. As shown in figure 6, $\langle D^{\text{eff}}\rangle / D^{\text{eff}}_{\text{nf}}$ approaches unity in the high-frequency regime. Compared to the cases shown in figure 6(a), a higher value of χ is required to reach the high-frequency limit in figure 6(b), due to the larger flow amplitude (Pe) in the latter. We note that $D^{\text{eff}}_{\text{nf}}$ depends explicitly on Pe_s. In the low-frequency regime, $\langle D^{\text{eff}}\rangle / D^{\text{eff}}_{\text{nf}}$ is lower for higher Pe_s because $D^{\text{eff}}_{\text{nf}}$ increases quadratically with the swim speed, whereas $\langle D^{\text{eff}}\rangle$ does not increase as rapidly [see figure 4(a)].

As shown in figure 6, the scaled dispersion coefficient $\left\langle D^{\text{eff}}\right\rangle / D^{\text{eff}}_{\text{nf}}$ for passive particles decreases monotonically with χ. For active particles, the scaled dispersion coefficient exhibits rich behavior that depends on the flow (Pe) and swim (Pe_s) speeds. At low activity [e.g., $Pe_s = 1$ in figure 6(a)], the scaled dispersion coefficient remains a monotonically decreasing function of χ. For higher activity [e.g., $Pe_s = 5$ in figure 6(a)], the scaled dispersion coefficient begins at a low plateau and increases as a function of χ, eventually converging to the common high-frequency limit.

Interestingly, in figure 6(b), the scaled dispersion coefficient exhibits oscillatory behavior as a function of χ for $Pe_s = 5$. This non-trivial variation occurs when the flow Péclet number is sufficiently large. For $Pe_s = 5$ and $Pe = 40$, we see that $\left\langle D^{\text{eff}}\right\rangle / D^{\text{eff}}_{\text{nf}}$ exhibits both a minimum and a maximum at different frequencies. The observed oscillatory behavior likely results from resonance in which the flow oscillation timescale matches an intrinsic timescale of the ABPs. Because the dynamics of ABPs involve multiple timescales, the intrinsic timescale that leads to resonant diffusion cannot be easily obtained. In the following, we investigate this phenomenon numerically by identifying the thresholds of this oscillation as a function of χ, Pe, and Pe_s. Physically, χ, Pe, and Pe_s characterize the flow oscillation timescale $1/\omega$, the flow advection timescale H/U_f, and the swimming timescale H/U_s, respectively.

We first examine the details of the oscillation in figure 7(a) by plotting $\langle D^{\text{eff}}\rangle / D^{\text{eff}}_{\text{nf}}$

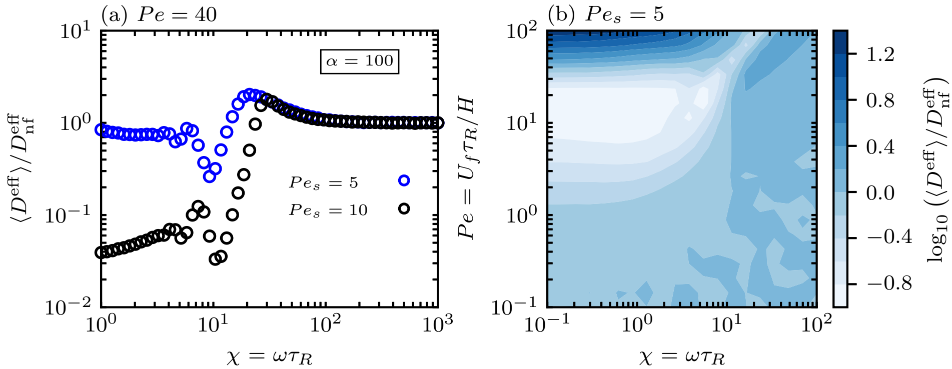

FIGURE 7. (a) Plots of $\langle D^{\text{eff}} \rangle / D^{\text{eff}}_{\text{nf}}$ as a function of χ for different values of Pe_s. (b) Contour plot of the logarithm of $\langle D^{\text{eff}} \rangle / D^{\text{eff}}_{\text{nf}}$ as a function of Pe and χ at $Pe_s = 5$. All results are from BD simulations with $\alpha = 100$, and $\gamma^2 = 0.1$. The contour plot is produced from a total of 400 data points, with 20 points uniformly spaced in logarithmic space along each axis.

as a function of χ for two values of Pe_s, using more data points than in figure 6. Even though it is not straightforward to determine the intrinsic timescale associated with resonant diffusion, one can rationalize its variation as a function of the swim speed. Compared to $Pe_s = 5$ (blue circles), the onset of oscillatory behavior of $\langle D^{\text{eff}} \rangle / D^{\text{eff}}_{\text{nf}}$ and the locations of its extrema shift to higher flow frequencies for higher activity ($Pe_s = 10$, black circles). This can be understood by considering the swim timescale, $\tau_s = H/U_s$, which characterizes the time it takes for the ABPs to traverse the channel in the transverse direction. As the swim speed (Pe_s) increases, τ_s decreases. Therefore, a smaller flow oscillation timescale (or higher flow oscillation frequency) is required to match the swim timescale.

Next, we consider how the oscillation in the dispersion coefficient further depends on the flow advection. In figure 7(b), we show a contour plot of the logarithm of $\langle D^{\text{eff}} \rangle / D^{\text{eff}}_{\text{nf}}$ as a function of Pe and χ for $Pe_s = 5$. We observe that as Pe increases, the extrema in $\langle D^{\text{eff}} \rangle / D^{\text{eff}}_{\text{nf}}$ shift to higher flow oscillation frequencies, reflected in the upward shift of the light-colored region in figure 7(b) with increasing χ. In addition to the swim timescale discussed in figure 7(a), the ABPs in the presence of flow also have a flow timescale defined by H/U_f. As Pe increases, this flow timescale decreases. To achieve resonance, the timescale defined by the flow oscillation, $1/\omega$, needs to be smaller. Due to the interplay of these timescales, in general $\langle D^{\text{eff}} \rangle / D^{\text{eff}}_{\text{nf}}$ exhibits non-monotonic behaviors as a function of Pe and χ. Furthermore, we note that the region where the scaled dispersion coefficient attains low values shrinks as Pe increases.

Together with the behavior of $\langle D^{\text{eff}} \rangle / D^{\text{eff}}_{\text{nf}}$ shown in figure 7(a), we note that increasing either Pe or Pe_s shifts the onset of oscillatory behavior to higher flow frequencies. This suggests that resonance can occur when the flow oscillation timescale matches some intrinsic timescale that results from the coupling between the swimming motion and the oscillatory flow advection. Formally, we have $\tau = \tau(Pe_s, Pe)$, where τ is the timescale that must match the flow oscillation timescale to achieve resonance. Extracting the functional dependence of τ on H/U_s and H/U_f is not pursued here. We note that resonant diffusion is commonly observed in both passive and active particle systems (Castiglione *et al.* 1998; Leahy *et al.* 2015; Khatri & Burada 2022; Chepizhko & Franosch 2022).

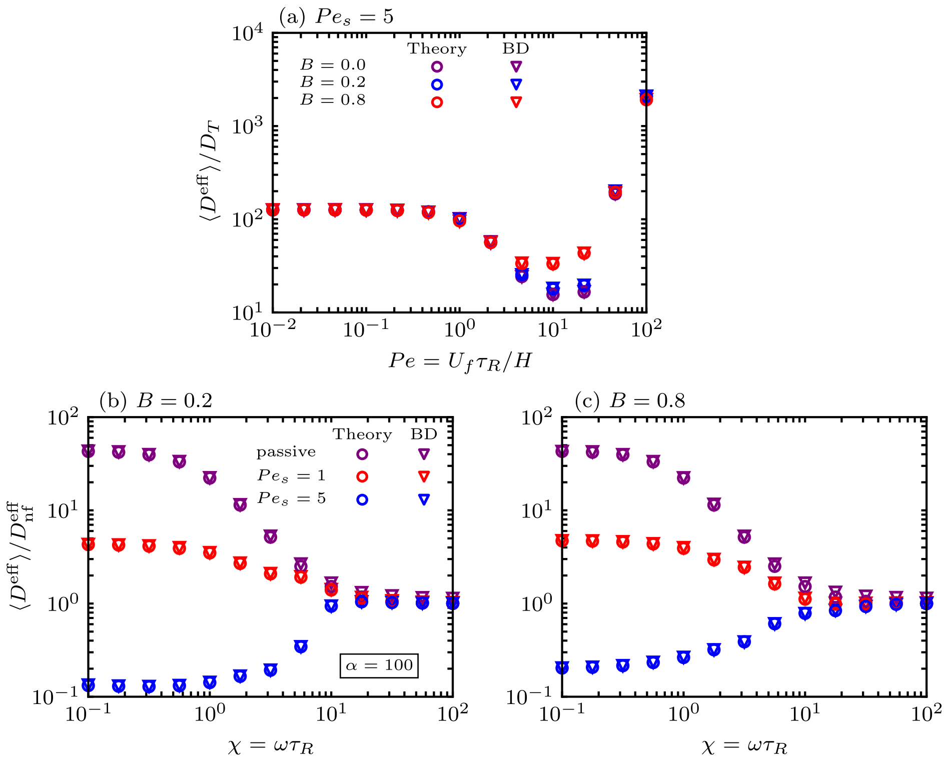

FIGURE 8. (a) Plots of $\langle D^{\text{eff}} \rangle / D_T$ as a function of Pe for different values of B. For all results in (a), $Pe_s = 5$, $\chi = 1$, $\gamma^2 = 0.1$, and $\kappa = 0.1$. (b) Plots of $\langle D^{\text{eff}} \rangle / D^{\text{eff}}_{\text{nf}}$ as a function of χ for $B = 0.2$. (c) Plots of $\langle D^{\text{eff}} \rangle / D^{\text{eff}}_{\text{nf}}$ as a function of χ for $B = 0.8$. For all results shown, circles represent results from numerical solutions of the full GTD theory, while triangles denote results from BD simulations. The labels shown in (b) also apply to the corresponding curves in (c). For (b) and (c), $Pe = 10, \alpha = 100$, and $\gamma^2 = 0.1$.

4.3. *Non-spherical particles*

We now analyze how the effective dispersivity is influenced by the shape of the particle. For an ellipsoidal particle, a shape factor is defined as $B = (r^2 - 1)/(r^2 + 1)$, where $r = a/b$. Here, a and b denote the lengths of the semi-major and semi-minor axes, respectively. For a sphere, $r = 1$ and $B = 0$. For a thin rod, we have $B \to 1$ as $r \to \infty$. Modeling the angular dynamics using Jeffery equation (Jeffery 1922), we have $\boldsymbol{\Omega}_f = \frac{1}{2} \nabla \times \boldsymbol{u}_f + B \boldsymbol{q} \times (\boldsymbol{E} \cdot \boldsymbol{q})$, where $E_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$ is the rate-of-strain tensor. In the oscillatory Poiseuille flow, we have

$$\Omega'^* = \Omega' \tau_D = \frac{(1-i) Pe}{2\kappa} (1 - B \cos 2\phi) \sinh\left((1+i)\kappa y^*\right) \operatorname{sech}\left((1+i)\kappa\right). \tag{4.1}$$

As in the spherical case, we assume a constant translational diffusivity and enforce the no-flux boundary condition (2.4).

In figure 8(a), we plot $\langle D^{\text{eff}} \rangle / D_T$ as a function Pe for different values of B. The vorticity term $(\nabla \times \boldsymbol{u}_f)$ in the angular velocity induces spinning on ABPs, which reduces their persistence and consequently their swim diffusion. For non-spherical particles $(B \neq 0)$, the additional alignment term from the rate-of-strain tensor is present. Because of this alignment, non-spherical particles lose less of their persistence. As a result, we observe that in figure 8(a), the minimum in the dispersion coefficient decreases as B decreases. As shown in figures 8(b) and 8(c), the scaled dispersion coefficient $\langle D^{\text{eff}}\rangle/D^{\text{eff}}_{\text{nf}}$ exhibits qualitatively similar behavior to that of spherical particles. However, the suppression in effective dispersivity is reduced, owing to the reduced spinning.

5. Concluding remarks

In this paper, we employed a GTD theory to study the longitudinal dispersion of ABPs in oscillatory Poiseuille flow. For passive particles, the time-averaged dispersion coefficient decreases monotonically with increasing flow oscillation frequency. As the frequency increases, Taylor dispersion is gradually suppressed due to the increasing oscillations of the flow. The long-time dispersion can be modeled as a random walk, from which a diffusivity is defined as $\ell^2_{\text{eff}}/\tau_{\text{eff}}$, where ℓ_{eff} is the step length and τ_{eff} is the de-correlation time. In the high-frequency limit, the step length ℓ_{eff} vanishes as a result of the rapid back-and-forth motion induced by the flow. Therefore, Taylor dispersion vanishes and $\langle D^{\text{eff}}\rangle \to D^{\text{eff}}_{\text{nf}} = D_T$ as $\chi \to \infty$. For active particles, we have shown that the high-frequency behavior is indistinguishable from that of passive particles when the scaled dispersion coefficient $\langle D^{\text{eff}}\rangle/D^{\text{eff}}_{\text{nf}}$ is considered. We note that for active particles $D^{\text{eff}}_{\text{nf}} = D_T + D^{\text{swim}} > D_T$.

We have shown that the effective dispersion coefficient of active particles can exhibit oscillatory behavior as a function of the flow frequency χ. When the external driving frequency (i.e., flow oscillation frequency) matches an intrinsic frequency, resonant diffusion can be observed. This distinct behavior of active particles results from the coupling between self-propulsion and oscillatory fluid advection. Without activity, resonant diffusion does not occur. Likewise, for active particles in a steady Poiseuille flow, no oscillatory dispersion arises due to the absence of a periodic driving force. In oscillatory Poiseuille flow, the oscillation frequency acts as an external control parameter that modulates particle dispersion. This modulation is particularly versatile for active particles, as flow oscillations can either enhance or suppress dispersion compared to the no-flow case.

In general, resonant or oscillatory dynamics may occur when multiple transport mechanisms are present. For example, the rotational dispersion coefficient of axisymmetric Brownian particles in oscillatory shear flows exhibits oscillatory behavior as a function of the flow frequency (Leahy *et al.* 2015). In this case, the natural frequency corresponds to the inverse of half a Jeffery orbit period, while the external frequency is the flow oscillation frequency. Resonant diffusion has also been observed in particle systems, including the diffusion of chiral particles in steady Poiseuille flow (Khatri & Burada 2022), gravitactic circle swimmers (Chepizhko & Franosch 2022), and particles in complex time-periodic flow fields with mean flow (Castiglione *et al.* 1998).

While the GTD theory applies to generic time-periodic flows, we have considered only the case where the driving pressure gradient consists of a single harmonic. An interesting extension would be to include a mean flow, in addition to the oscillatory component that averages to zero. Particle transport due to the interaction between the steady and oscillatory components of the flow may lead to qualitatively different dispersion behavior. In particular, it would be interesting to examine how the oscillatory dispersion behavior is modified. While our analysis focuses on flows in planar channels, the GTD theory can be generalized to corrugated channels and periodic porous media (Peng 2024; Alonso-Matilla *et al.* 2019). For example, it would be interesting to examine the transport behavior of active particles in peristaltic flow (Chakrabarti & Saintillan 2020).

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Declaration of interests

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Appendix A. The passive solution

The displacement field at $O(1)$ is governed by

$$\frac{\partial b_0^*}{\partial t^*} + \frac{\partial}{\partial y^*}\left(-\gamma^2 \frac{\partial b_0^*}{\partial y^*}\right) + \frac{\partial}{\partial \phi}\left(\Omega_f^* b_0^* - \frac{\partial b_0^*}{\partial \phi}\right) = \left(U_0^{\mathrm{eff}*} - u^*\right) g_0, \tag{A 1a}$$

$$-\frac{\partial b_0^*}{\partial y^*} = 0, \quad \text{at} \quad y^* = \pm 1, \tag{A 1b}$$

$$\int_{-1}^{1} \mathrm{d}y^* \int_{\mathbb{S}} b_0^* \mathrm{d}\boldsymbol{q} = 0, \tag{A 1c}$$

which admits a solution of the form $b_0^* = \mathrm{Re}[A_0'(y^*)e^{i\chi t^*}/(2\pi)]$. Here A_0' satisfies

$$i\chi A_0' - \gamma^2 \frac{\mathrm{d}^2 A_0'}{\mathrm{d}y^{*2}} = \overline{u'^*} - u'^*, \tag{A 2a}$$

$$-\frac{\mathrm{d}A_0'}{\mathrm{d}y^*} = 0, \quad \text{at} \quad y^* = \pm 1, \tag{A 2b}$$

$$\int_{-1}^{1} A_0' \mathrm{d}y^* = 0. \tag{A 2c}$$

One can show that the solution is given by

$$A_0'(y^*) = \alpha_0 + \alpha_1 \cosh\left((1+i)\frac{\sqrt{\chi}}{\sqrt{2}\gamma} y^*\right) + \alpha_2 \cosh\left((1+i)\kappa y^*\right), \tag{A 3}$$

where

$$\alpha_0 = \frac{Pe(1-i)}{2\kappa^3 \chi} \tanh\left((1+i)\kappa\right), \tag{A 4a}$$

$$\alpha_1 = -\frac{\sqrt{2} Pe\, \gamma}{\chi^{1/2}\kappa\left(2\kappa^2\gamma^2 - \chi\right)} \frac{\tanh\left((1+i)\kappa\right)}{\sinh(1+i)\frac{\sqrt{\chi}}{\sqrt{2}\gamma}}, \tag{A 4b}$$

$$\alpha_2 = \frac{Pe}{\kappa^2\left(2\kappa^2\gamma^2 - \chi\right)} \operatorname{sech}\left((1+i)\kappa\right). \tag{A 4c}$$

One interesting limit is $\kappa \to 0$, where the viscous length scale, $\sqrt{2\nu/\omega}$, is much larger than the channel half-width, H. As $\kappa \to 0$, we have

$$\alpha_0 = \frac{Pe}{\chi\kappa^2} + O(\kappa^2), \quad \alpha_2 = -\frac{Pe}{\chi\kappa^2} + O(\kappa^2), \tag{A 5a}$$

$$\alpha_1 = \frac{(1+i)\sqrt{2}Pe}{\chi^{3/2}} \frac{1}{\sinh\left((1+i)\frac{\sqrt{\chi}}{\sqrt{2}\gamma}\right)} + O(\kappa^2), \tag{A 5b}$$

$$\cosh\left((1+i)\kappa y^*\right) = 1 + O(\kappa^2). \tag{A 5c}$$

Therefore, the singular contributions from α_0 and $\alpha_2 \cosh[(1+i)\kappa y^*]$ in (A 3) are canceled out while the second term in (A 3) is regular. Overall $A_0'(y^*)$ is finite as $\kappa \to 0$,

$$A_0'(y^*) = \frac{Pe}{3\chi^2}\left(-6\gamma^2 + i\left(1-3y^{*2}\right)\chi\right) + \frac{\sqrt{2}Pe\,\gamma\,(1+i)}{\chi^{3/2}} \frac{\cosh\left((1+i)\frac{\sqrt{\chi}}{\sqrt{2}\gamma}y^*\right)}{\sinh\left((1+i)\frac{\sqrt{\chi}}{\sqrt{2}\gamma}\right)} + O(\kappa^2). \tag{A 6}$$

Another limit that we examine is $\chi \to 0$. In this limit, we have

$$\alpha_0 = \left(\frac{1}{2} - \frac{i}{2}\right)\frac{Pe}{\kappa^3\chi}\tanh\left((1+i)\kappa\right) + O(\chi^3), \tag{A 7a}$$

$$\alpha_1 = -\left(\frac{1}{2} - \frac{i}{2}\right)\frac{Pe}{\kappa^3\chi}\tanh\left((1+i)\kappa\right) + O(\chi), \tag{A 7b}$$

$$\alpha_2 = \frac{Pe}{2\gamma^2\kappa^4}\,\mathrm{sech}\left((1+i)\kappa\right) + O(\chi), \tag{A 7c}$$

$$\cosh\left((1+i)\frac{\sqrt{\chi}}{\sqrt{2}\gamma}y^*\right) = 1 + O(\chi). \tag{A 7d}$$

The singular contributions from α_0 and $\alpha_1 \cosh\left((1+i)\frac{\sqrt{\chi}}{\sqrt{2}\gamma}y^*\right)$ in (A 3) are canceled out while the third term in (A 3) is regular. Overall $A_0'(y^*)$ is finite as $\chi \to 0$,

$$A_0'(y^*) = \frac{Pe}{2\gamma^2\kappa^4}\frac{\cosh\left((1+i)y^*\kappa\right)}{\cos\left((1+i)\kappa\right)} + \frac{(1+i)Pe}{12\gamma^2\kappa^5}\left[3i + \left(1-3y^{*2}\right)\kappa^2\right]\tanh\left((1+i)\kappa\right) + O(\chi). \tag{A 8}$$

The last limit that we consider is $\left(2\kappa^2\gamma^2 - \chi\right) \to 0$. We define $\epsilon = \left(2\kappa^2\gamma^2 - \chi\right)$. This limit $\epsilon \to 0$ is of particular interest when we look at (A 4b) and (A 4c). We show that in this limit,

$$\alpha_0 = \left(\frac{1}{4} - \frac{i}{4}\right)\frac{Pe}{\gamma^2\kappa^5}\tanh\left((1+i)\kappa\right) + O(\epsilon), \tag{A 9a}$$

$$\alpha_1 = -\frac{Pe}{\kappa^2\epsilon}\,\mathrm{sech}\left((1+i)\kappa\right) + O(\epsilon), \tag{A 9b}$$

$$\alpha_2 = \frac{Pe}{\kappa^2\epsilon}\,\mathrm{sech}\left((1+i)\kappa\right) + O(\epsilon^3), \tag{A 9c}$$

$$\cosh\left((1+i)\frac{\sqrt{\chi}}{\sqrt{2}\gamma}y^*\right) = \cosh\left((1+i)\kappa y^*\right) + O(\epsilon). \tag{A 9d}$$

The singular contributions from $\alpha_1 \cosh\left((1+i)\frac{\sqrt{\chi}}{\sqrt{2}\gamma}y^*\right)$ and $\alpha_2\cosh\left((1+i)\kappa y^*\right)$ are canceled out while the first term in (A 3) is regular. This shows in the limit $\epsilon \to 0$, $A_0'(y^*)$ is finite, and $A_0'(y^*)$ takes the form

$$A_0'(y^*) = \eta_0 + \eta_1 \cosh\left((1+i)y^*\kappa\right) + \eta_2 \sinh\left((1+i)y^*\kappa\right) + O(\epsilon), \tag{A 10}$$

where

$$\eta_0 = \frac{(1-i)Pe}{4\gamma^2\kappa^5}\tanh\left((1+i)\kappa\right) - \frac{Pe}{4\gamma^2\kappa^4}\frac{1}{\sinh\left((1+i)\kappa\right)}, \tag{A 11a}$$

$$\eta_1 = 1 + (1+i)\kappa\coth\left((1+i)\kappa\right), \tag{A 11b}$$

$$\eta_2 = -y^*(1+i)\kappa\tanh\left((1+i)\kappa\right). \tag{A 11c}$$

Appendix B. The high-frequency limit

Here we analyze the governing equations (2.27) and (2.30) in the high-frequency limit characterized by $\chi \gg 1$ while keeping the ratio $\chi/\kappa^2 = \alpha = 2\nu\tau_R/H$ constant. That is, $\alpha = O(1)$ as $\chi \to \infty$. We also assume that all other non-dimensional parameters are $O(1)$ as $\chi \to \infty$. To facilitate our analysis, we write the long-time solution to g as a Fourier series,

$$g(y^*, \phi, t^*) = \sum_{n=-\infty}^{+\infty} e^{in\chi t^*} g_n(y^*, \phi). \tag{B 1}$$

Inserting this expansion into equation (2.27), we obtain

$$in\chi g_n + Pe_s \sin\phi\,\frac{\partial g_n}{\partial y^*} - \gamma^2\frac{\partial^2 g_n}{\partial y^{*2}} + \frac{\partial}{\partial\phi}\left[\mathrm{Re}\left(\Omega'^*\right)\frac{1}{2}\left(g_{n-1}+g_{n+1}\right) - \mathrm{Im}\left(\Omega'^*\right)\frac{1}{2i}\left(g_{n-1}-g_{n+1}\right)\right] - \frac{\partial^2 g_n}{\partial\phi^2} = 0. \tag{B 2}$$

The conservation condition becomes

$$\frac{1}{2}\int_{-1}^{1}\mathrm{d}y^*\int_0^{2\pi}\mathrm{d}\phi\sum_{n=-\infty}^{+\infty} e^{in\chi t^*} g_n(y^*, \phi) = 1. \tag{B 3}$$

Making use of orthogonality, we have

$$\frac{1}{2}\int_{-1}^{1}\mathrm{d}y^*\int_0^{2\pi}\mathrm{d}\phi\, g_0(y^*,\phi) = 1, \quad \text{and} \quad \int_{-1}^{1}\mathrm{d}y^*\int_0^{2\pi}\mathrm{d}\phi\, g_n(y^*,\phi) = 0, n \neq 0. \tag{B 4}$$

In the high-frequency limit, one can show that $g_0 = O(1)$ and $g_1 = o(1)$. At leading order, we have

$$Pe_s\sin\phi\,\frac{\partial g_0}{\partial y^*} - \gamma^2\frac{\partial^2 g_0}{\partial y^{*2}} - \frac{\partial^2 g_0}{\partial\phi^2} = 0. \tag{B 5}$$

The no-flux condition is given by

$$Pe_s\sin\phi\, g_0 - \gamma^2\frac{\partial g_0}{\partial y^*} = 0, \quad y^* = \pm 1. \tag{B 6}$$

This shows that, at high frequencies, the governing equation for g, to leading order, reduces to that of ABPs in a channel without flow. Similarly, one can show that the displacement field also satisfies the equation without flow. As a result, the effective dispersion coefficient approaches the no-flow result in the high-frequency limit.

Appendix C. Brownian dynamics

The discretized Langevin equations are given by

$$x_{n+1} = x_n + u_f\left(y_n, t_n\right) \Delta t + U_s \cos\left(\phi_n\right) \Delta t + \Delta x^B, \tag{C 1a}$$

$$y_{n+1} = y_n + U_s \sin\left(\phi_n\right) \Delta t + \Delta y^B, \tag{C 1b}$$

$$\phi_{n+1} = \phi_n + \Omega_f(y_n, \phi_n, t_n)\Delta t + \Delta\phi^B, \tag{C 1c}$$

where Δt is the time step. The Brownian displacements Δx^B, Δy^B, and $\Delta\phi^B$ are sampled from independent white noise processes. The translational Brownian displacement has a variance of $2D_T\Delta t$, and the rotary Brownian displacement has a variance of $2\Delta t/\tau_R$. A potential-free algorithm is used to implement the no-flux condition (Heyes & Melrose 1993). For all simulations, a sufficiently small time step is used to resolve all the physical timescales in the system. The total simulation time is long enough to ensure convergence to the long-time behavior. To ensure good statistics, all simulations are performed with $200,000$ particles.

Appendix D. Numerical simulation

The governing equations (2.27) and (2.30) are solved numerically using Dedalus (Burns *et al.* 2020). The physical space (y^*) is discretized on a Chebyshev grid with 128 nodes, and the orientational space is represented in Fourier space with 128 nodes. For time integration, we employ a second-order Crank-Nicolson-Adams-Bashforth scheme.

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